Spatial extremes approach for waves in Gulf of Mexico

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Motivation: Extreme waves of hurricane-dominated regions

- Tropical cyclone is **rare and severe**, and are relatively small scale by comparison to extra-tropical storms and hurricane track has an important influence on severity of sea state at a particular location.
- Extreme wave estimation is always accompanied with **large uncertainty,** especially large in hurricane-dominated region.
- From a structural design perspective, broad-brush application of a design safety factor may be inappropriate.
- Gulf of Mexico has long history in offshore oil & gas with strong knowledge foundation.

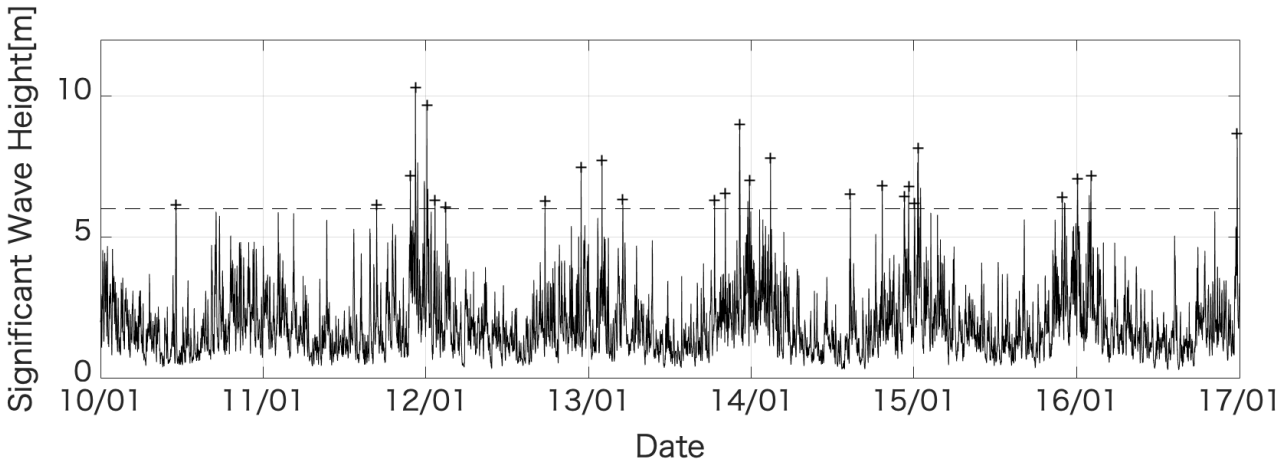
"Can spatial extremes model improve extreme wave estimation in GoM?"

* Jonathan and Ewans (2007)

Classic approach to extreme wave estimation

Extreme value theory: Limiting extreme value distributions for asymptotic behavior for extreme values from n -> ∞ (Fisher & Tippett, 1928, Gumbel, 1952)

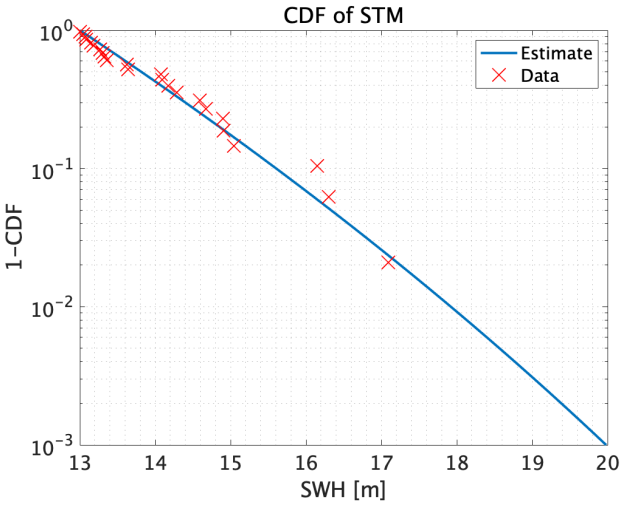
Extract Peak-over-threshold data at a given location



Fit Generalized Pareto distribution

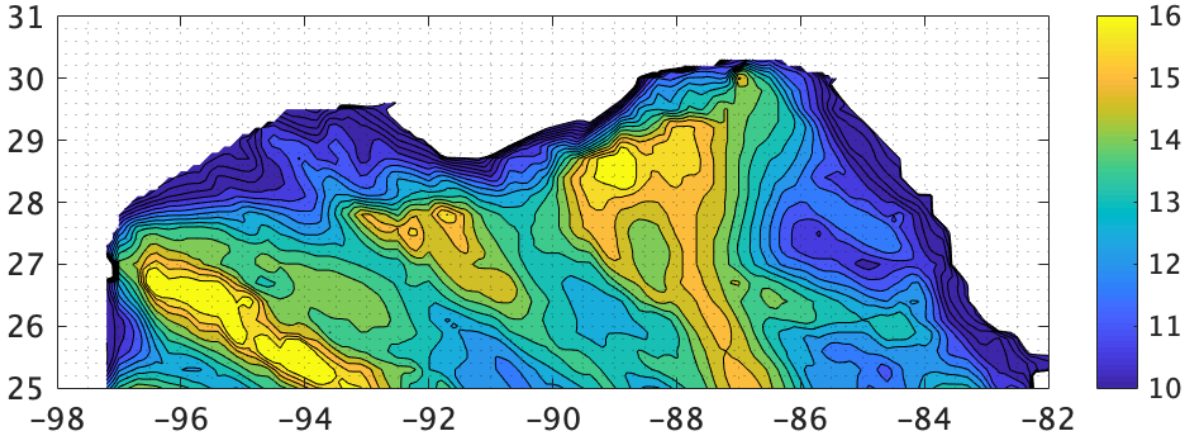
$$F(x) = 1 - (1 + \xi(x-\mu)/\sigma)^{-1/\xi} \qquad \xi \neq 0$$

$$F(x) = 1 - \exp(-(x-\mu)/\sigma)$$

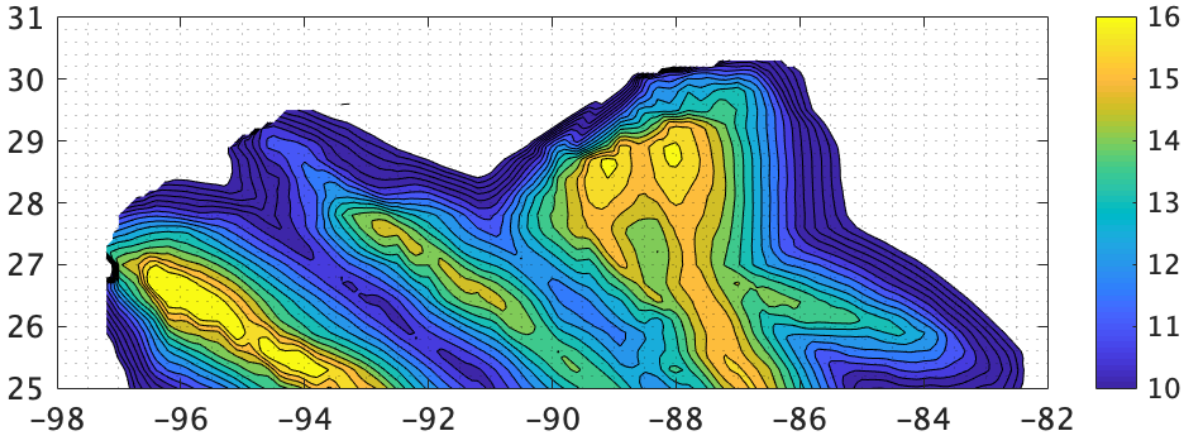


Per location approach vs Spatial Extremes approach

100-year return values from per location analysis



Footprint of 6 largest hurricanes during 1900-2008



- **100-year return period value from per location approach is dominated by the very few strong hurricanes (only 6 during 100 years) that approached the region**
- **How important is the footprint of 6 largest hurricanes, when**
  - **The hurricane tracks are realization from population**
  - **We have more records of hurricane tracks to learn from**
- **Data from "per location approach" is mostly a fraction of the peak value.**

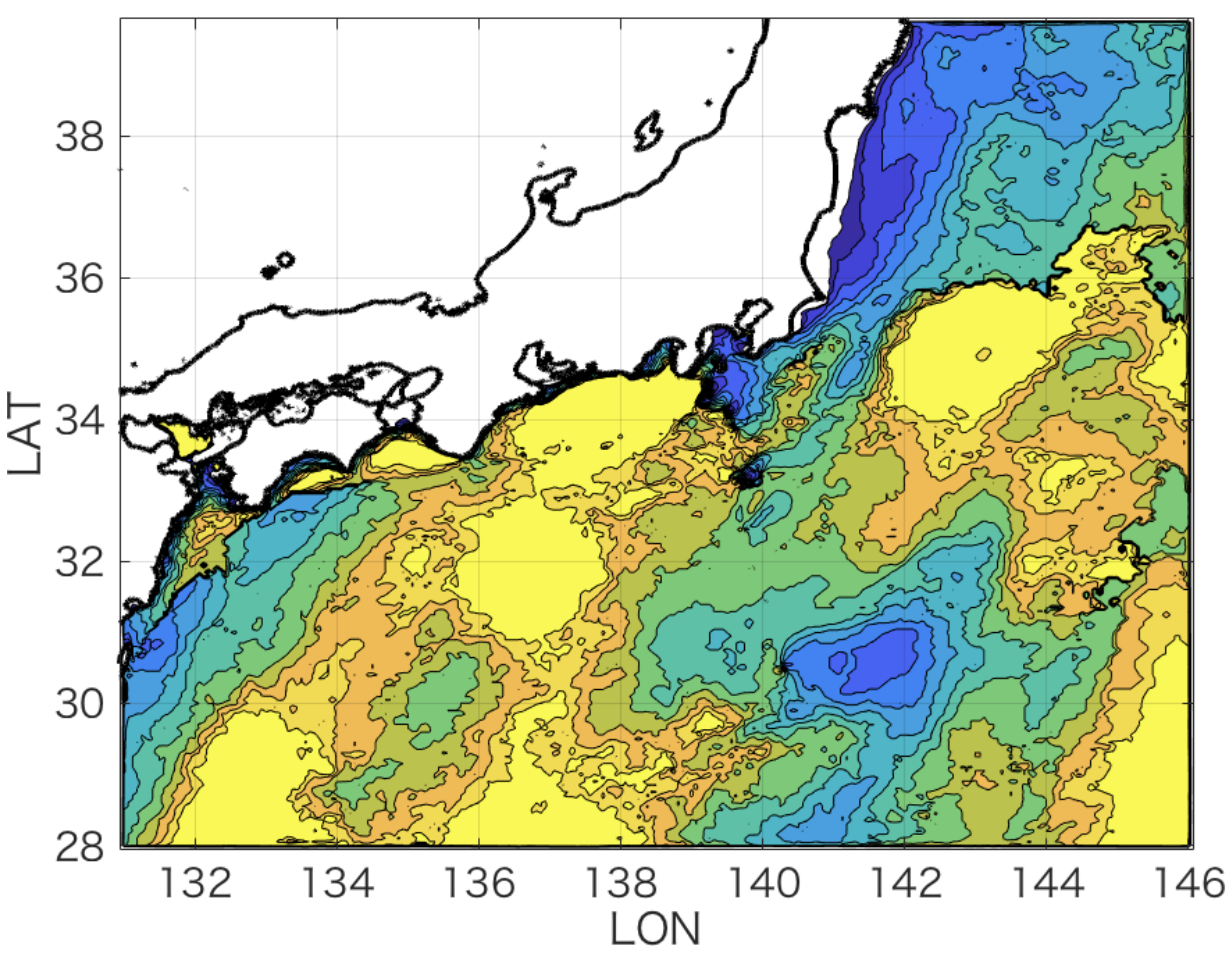
Methodology for spatial extremes

Previous studies: Techniques for utilizing spatial data

- Spatial pooling (Heideman and Mitchell, 2009)
  - Increase the sample size for analysis by including events from multiple locations in a neighbourhood;
  - Problematic since a given cyclone event at neighbouring locations produces dependent extremes, violating a basic assumption that independent observations be used.
- Cyclone track-shifting, Explicit track modelling (Vickery et al., 2000, 2010)
  - Track-shifting and empirical track modelling produce large numbers of realisations of cyclones over the region, thus increasing sample size;
  - the quality of resulting return value estimates depends on exactly how physically realistic the track-shifting or track modelling algorithms are.
  - The statistics literature provides methods to estimate spatially-dependent extremes using extensions of univariate extreme value theory (e.g. Davison et al., 2012; Ross et al., 2017a); but these constitute overly complicated models for situations where sample size is small

Spatial Extremes: “STM-E Approach” [Wada et al, 2018]

Classical approach: *Statistics per location*



100yr RP value Hs

Per location approach gives spatially ROUGH estimation
Based on 21years wave simulation data

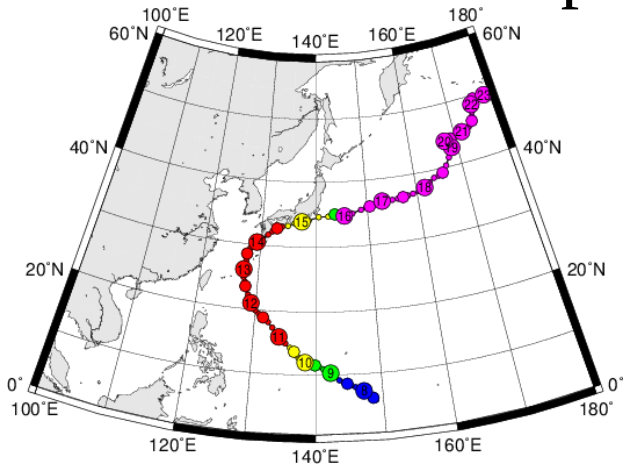
Utilize spatial dependency based on footprint of largest Hs during hurricanes

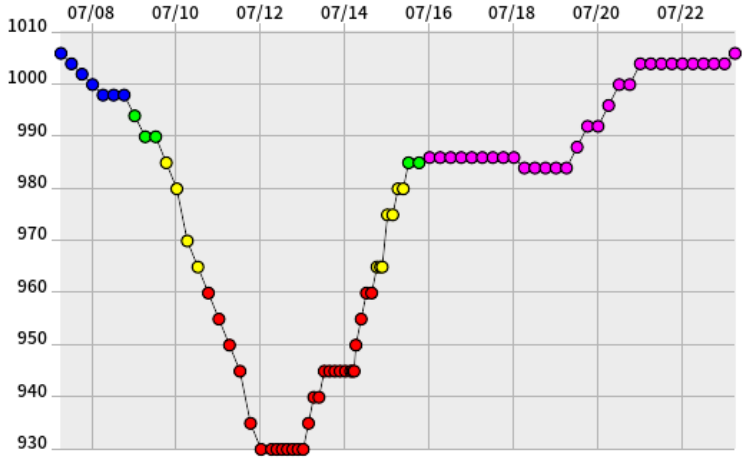
Wada, R., Waseda, T., & Jonathan, P. (2018). A simple spatial model for extreme tropical cyclone seas. Ocean Engineering, Vol 169, 315-325.

What is STM-E Approach?

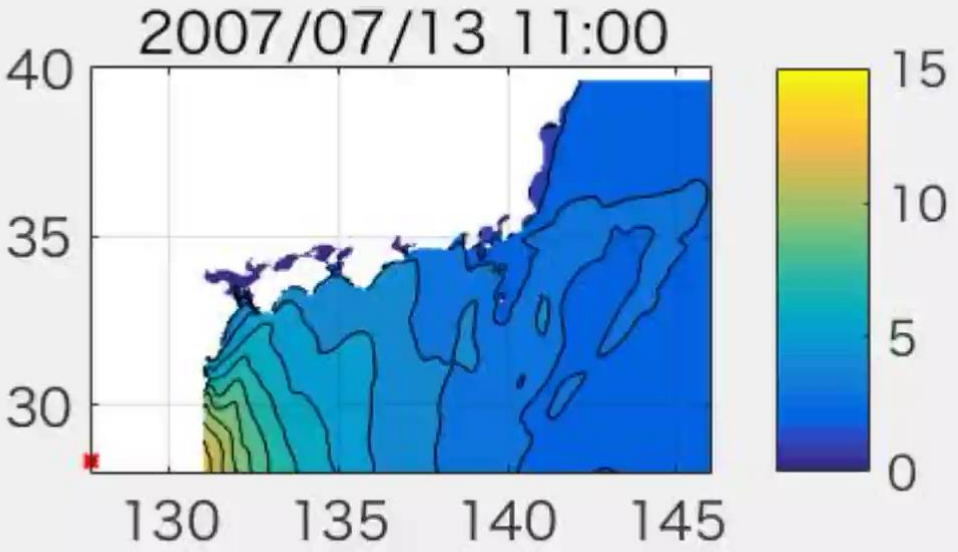
Example 1: Extreme waves from Tropical Cyclones (Jul 2007)

July 2007
(200704)

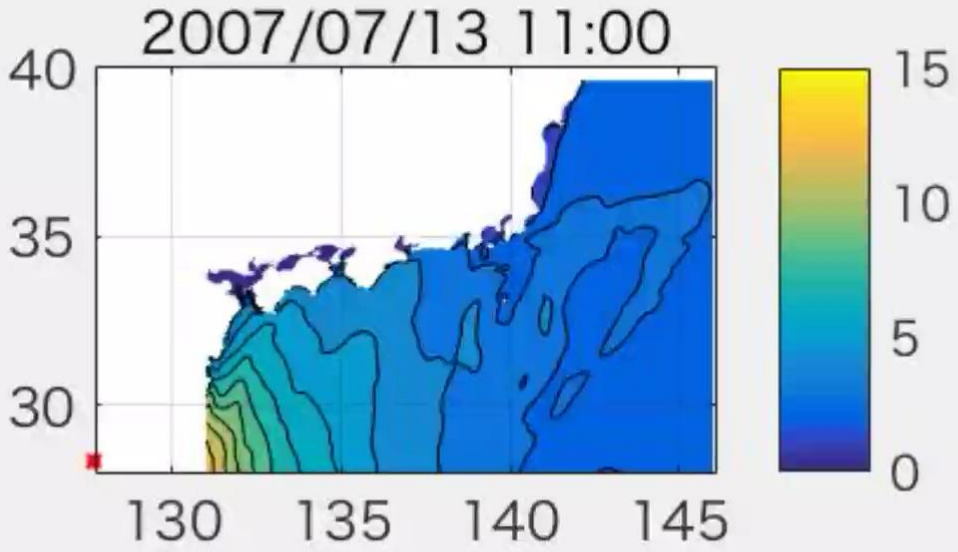




Snapshots of Hs during tropical cyclone



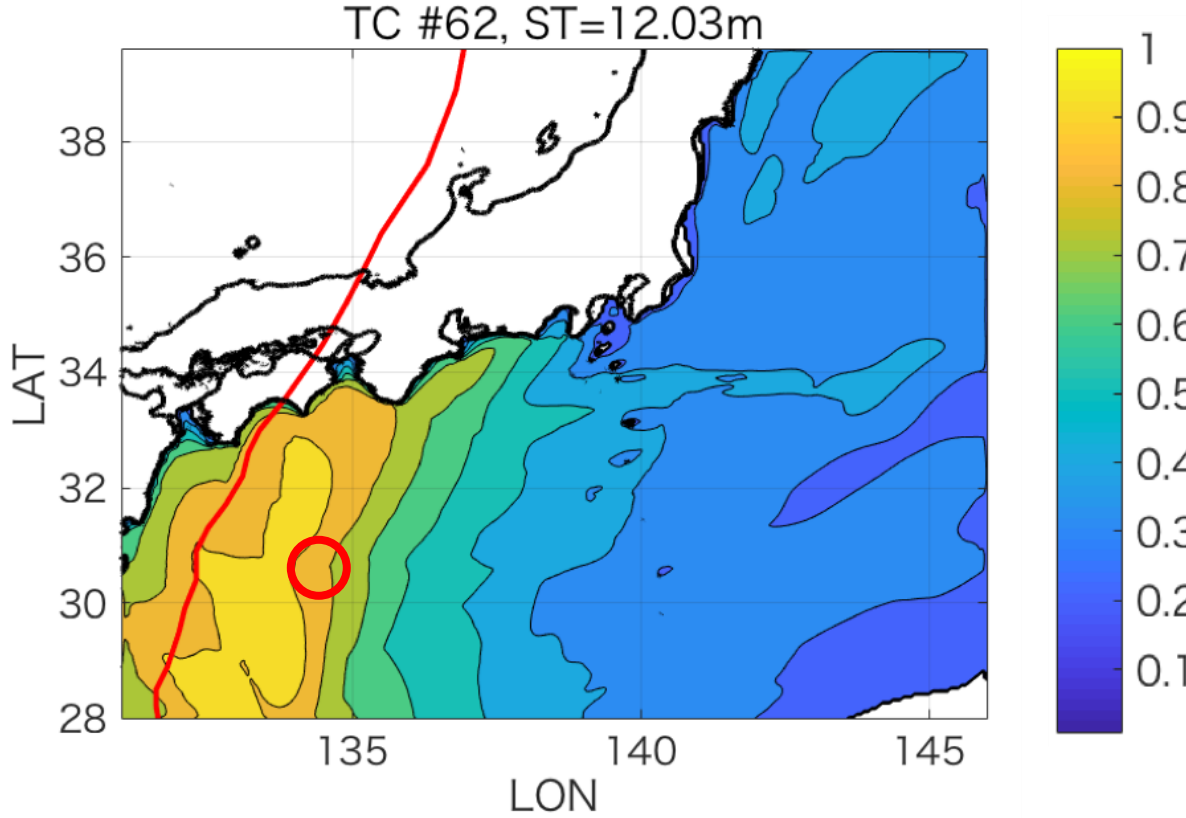
Footprints of largest Hs during tropical cyclone



What is STM-E Approach?

For each hurricane event, we extract

- Space-Time Maximum (STM)
  The largest significant wave height observed during the hurricane event over the whole region
- Exposure (for each location, [0,1])
  How large the largest Hs at each location was compared to STM.
  Absolute position not relative.



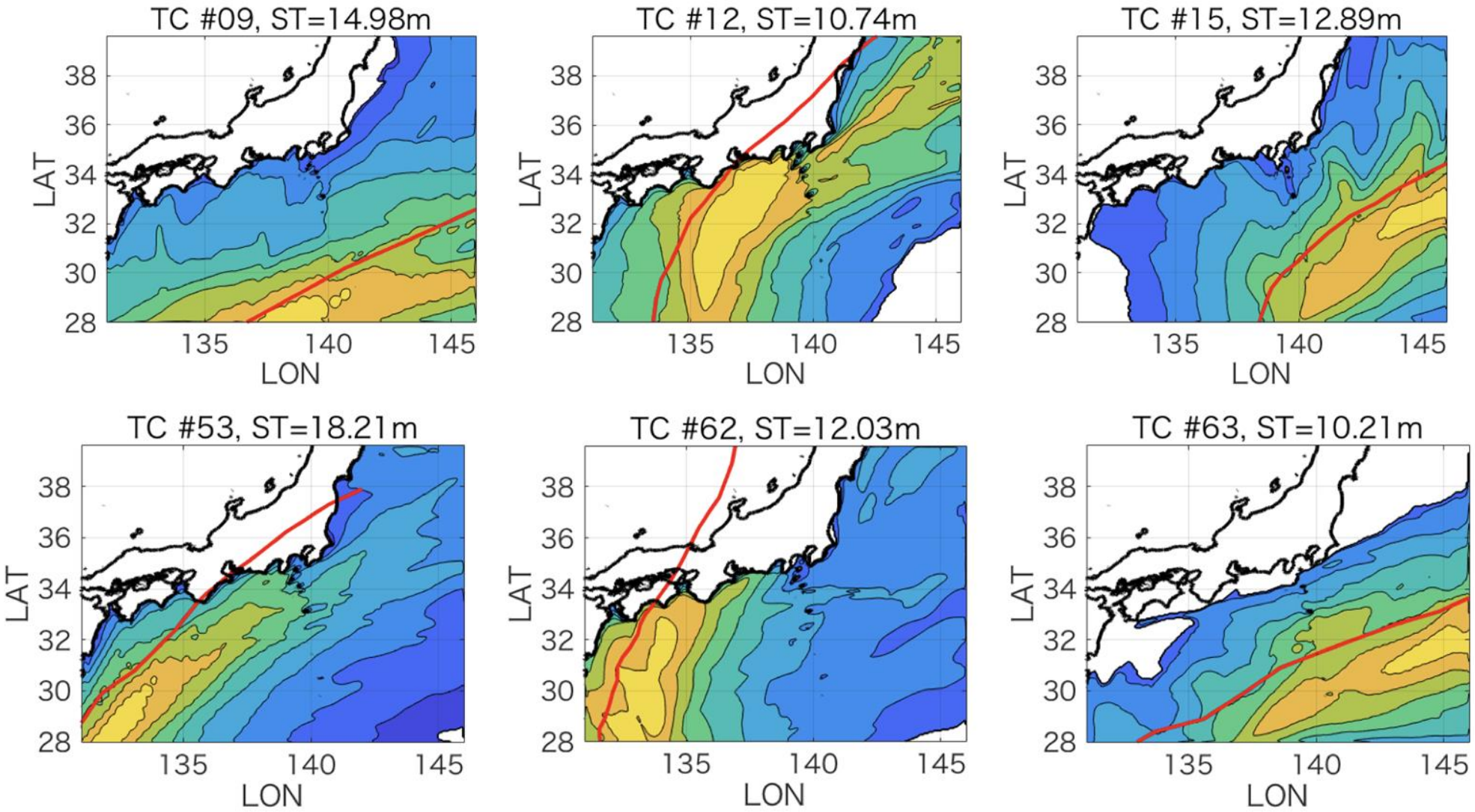
- Learn from the Hs footprint of past hurricanes
- No need to run additional wave simulations

Example: TC #62
- STM = 12.03 [m]
- Exposure 0~1 distribution provided above

Wada, R., Waseda, T., & Jonathan, P. (2018). A simple spatial model for extreme tropical cyclone seas. Ocean Engineering, Vol 169, 315-325.

STM-E Approach formulation

Example of obtained data set of STM & Exposure in sea around Japan

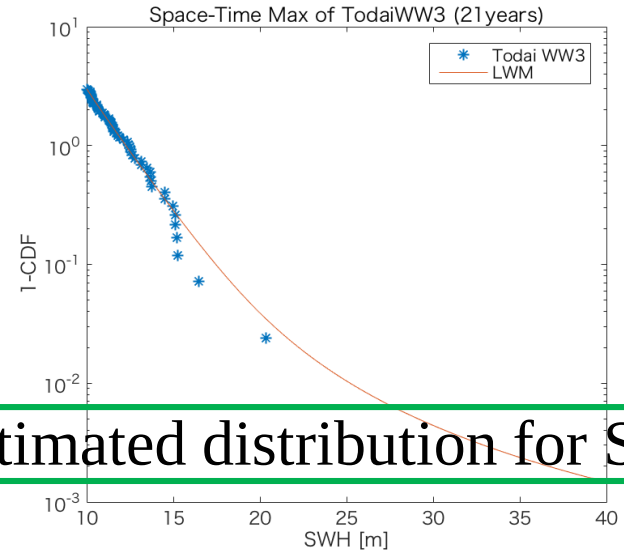


Wada, R., Waseda, T., & Jonathan, P. (2018). A simple spatial model for extreme tropical cyclone seas. Ocean Engineering, Vol 169, 315-325.

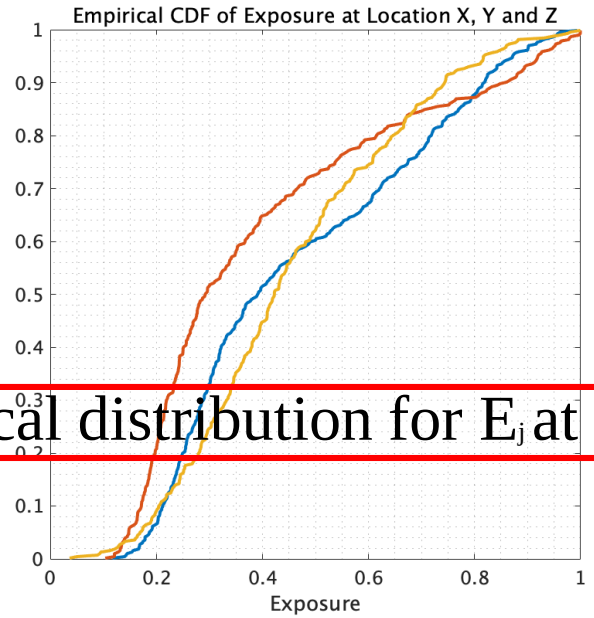
STM-E Approach formulation

Combine STM & Exposure to generate extreme wave distribution for each location

$$
\begin{aligned}
F_{H_j}(h) &= \mathbb{P}(H_j \le h) \\
&= \int_s \mathbb{P}(E_j S \le h | S = s) f_S(s) ds \\
&= \int_s \mathbb{P}(E_j \le h/s) f_S(s) ds \\
&= \int_s F_{E_j}(h/s) f_S(s) ds
\end{aligned}
$$



F_S : estimated distribution for STM



F_{Ej} : empirical distribution for E_j at location j

F_{Hj} : distribution of storm severity H_j at location j

$H_j = E_j$ (Exposure) x STM

Assuming the two distributions are independent

Kendall's rank correlation

Hurricane	STM	STM rank	Exposure at position X	Exposure rank
#1	12.1m	3	0.7	3
#2	9.7m	4	0.9	1
#3	14.2m	2	0.3	4
#4	16.1m	1	0.8	2
#5	8.2m	5	0.2	5

Kendall's tau statistic for the "rank correlation" between "STM" and "Exposure at each location"

$$\tau_j = \frac{1}{n_S(n_S-1)} \sum_{i \neq k} \mathrm{sgn}(s_i - s_k)\mathrm{sgn}(e_{ij} - e_{kj})$$

Gaussian-distributed with zero mean and variance $2(2n_S+5)/(9n_S(n_S-1)$ provides a means of identifying unusual values of τ which may indicate dependence between S and E_j.

Wada, R., Waseda, T., & Jonathan, P. (2018). A simple spatial model for extreme tropical cyclone seas. Ocean Engineering, Vol 169, 315-325.

STM-E Approach

$$F_{H_j}(h) = \int_s F_{E_j}(h/s) f_S(s) ds$$

Spatial empirical interpolation

& Temporal extrapolation

F_{Hj} : distribution of storm severity H_j at location j

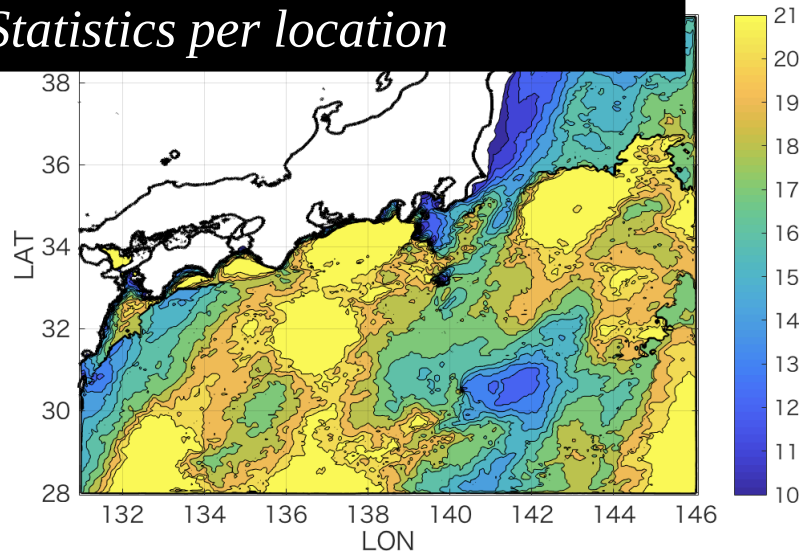
$H_j = E_j$ (Exposure) x STM

F_{Ej} : empirical distribution for E_j at location j

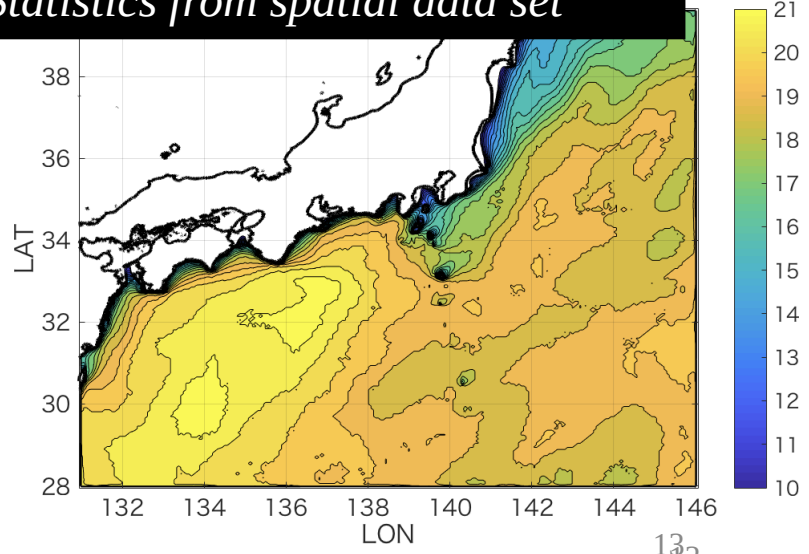
F_S : estimated distribution for STM

- **Smooth extreme wave estimation with smaller variance**
- **Learn from the Hs footprint of past hurricanes**
- **No need to run additional wave simulations**

Classical approach
Statistics per location



STM-E (Wada et al, 2018)
Statistics from spatial data set



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Application to the Gulf of Mexico

GOMOS 08 Data set

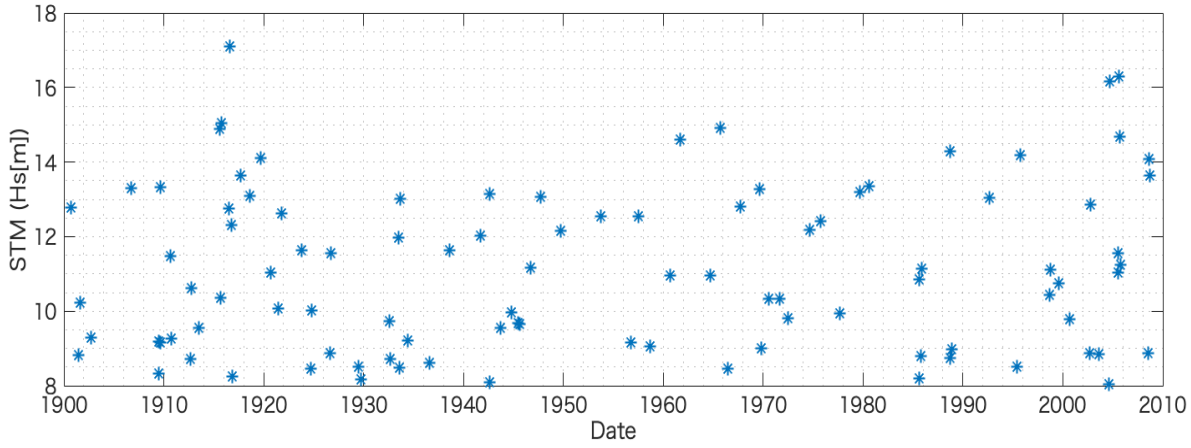
- A comprehensive metocean study for the Gulf of Mexico made by *Oceanweather*.
- Wave data based on 3rd generation wave model
  - with 1/16th degree grid (7km) resolution
  - from 1900 to 2008 with 379 tropical events (tropical storm and hurricane, of all significant storm events)

[1] GOMOS, Gulf of Mexico hindcast study, GOMOS08, Oceanweather Inc. 2008.

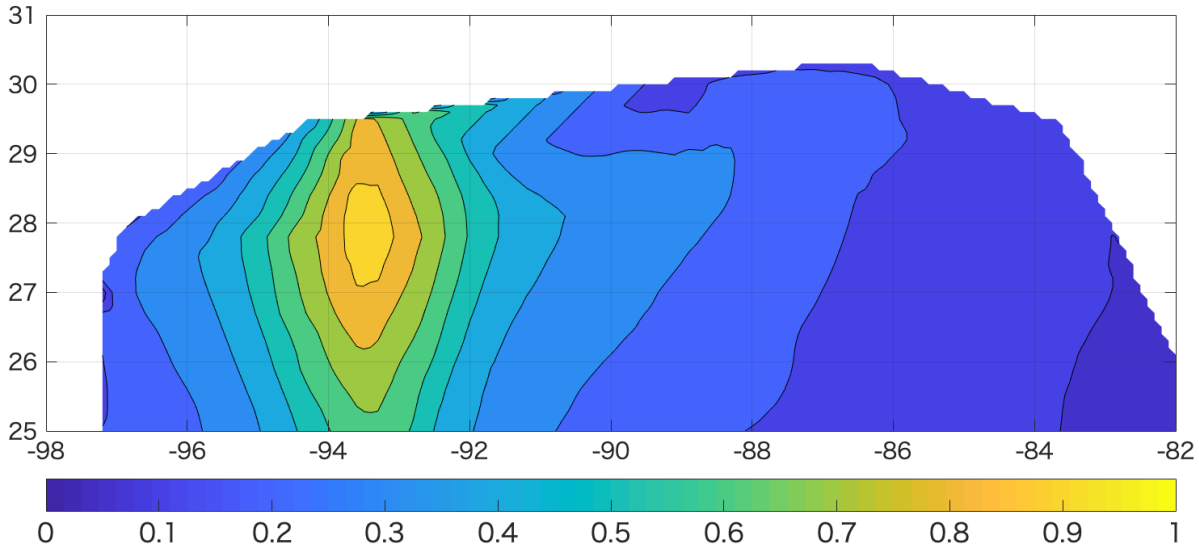
Extracting STM and Exposure

Set a threshold for STM and extract,

- **STM (Space Time Maxima) for each TC**
  Largest Hs during the storm in the whole GOMOS 08 region
- **Exposure for each location and each TC**
  0-1 value defined by the location maximum during TC / STM

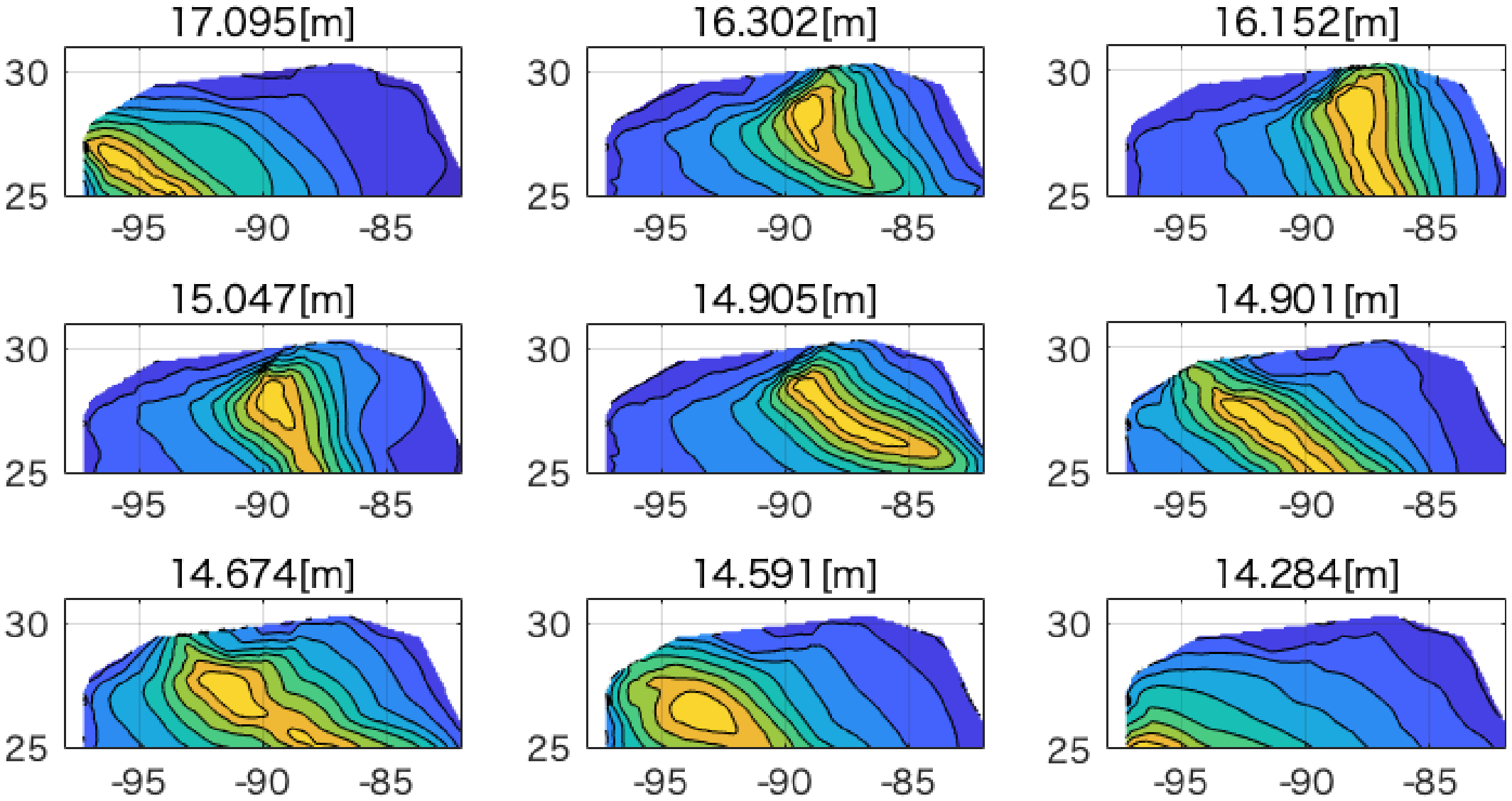


Time series of STM over 8m (1900-2008)

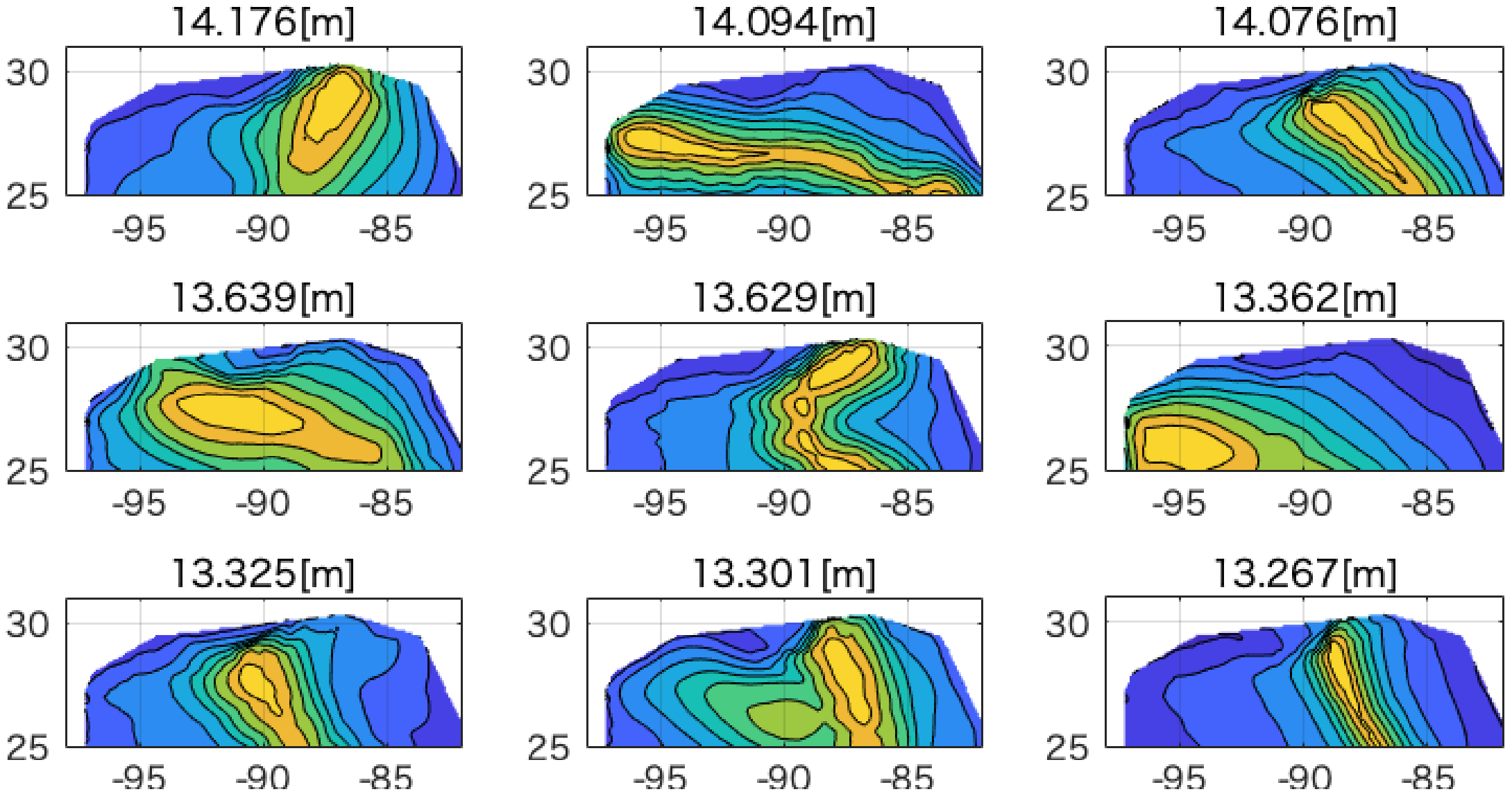


Sample of Exposure during a Tropical Cyclone

Exposure (for Top 1~9 STM)



Exposure (for Top 10~18 STM)

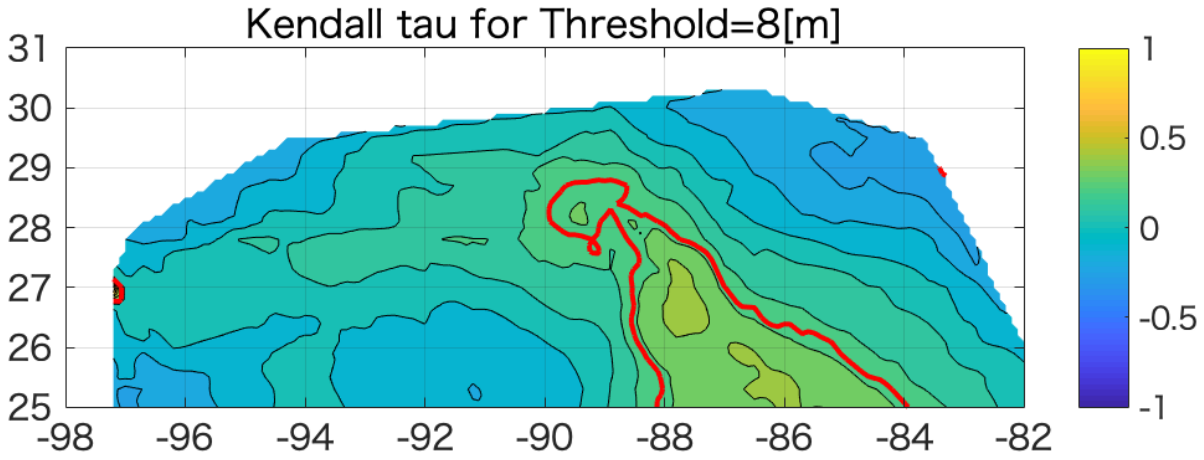


Dependency of STM and Exposure? Unfortunately, YES

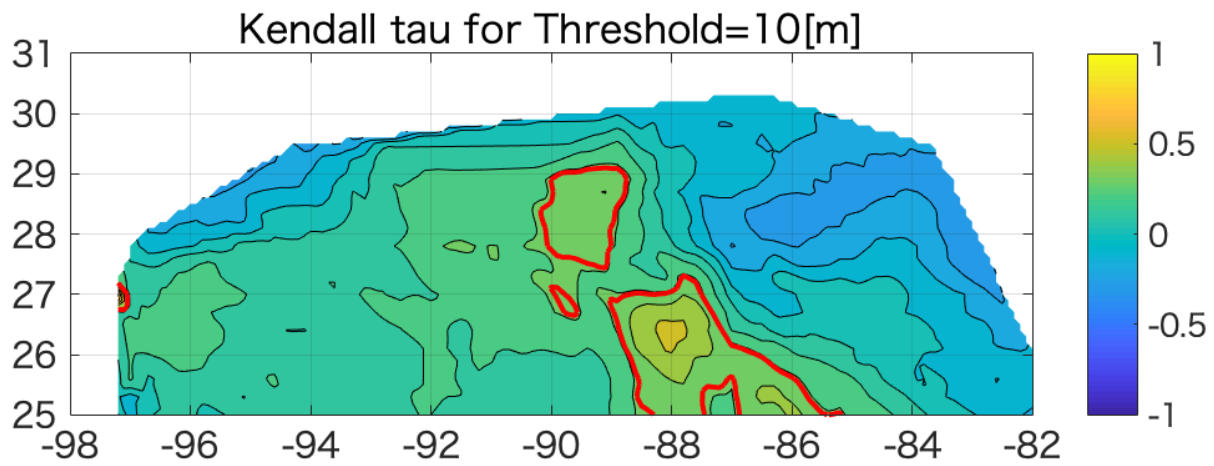
Kendall's tau statistic for the "rank correlation" between "STM" and "Exposure at each location"

$$\tau_j = \frac{1}{n_S(n_S-1)} \sum_{i \neq k} \mathbf{sgn}(s_i - s_k)\mathbf{sgn}(e_{ij} - e_{kj})$$

Gaussian-distributed with zero mean and variance $2(2n_S+5)/(9n_S(n_S-1)$ provides a means of identifying unusual values of τ which may indicate dependence between S and E_j.



Kendall's tau result for thresholds of 8m

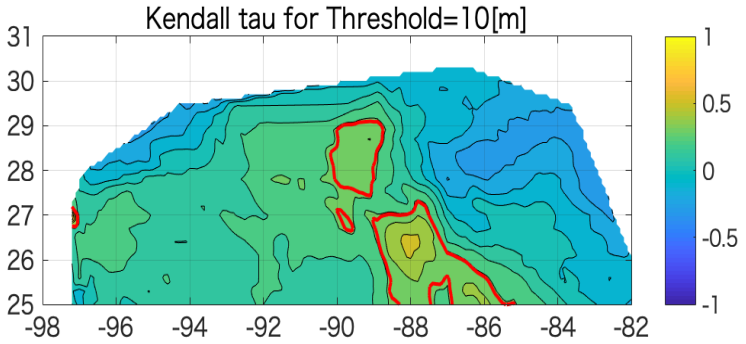


Kendall's tau result for thresholds of 10m

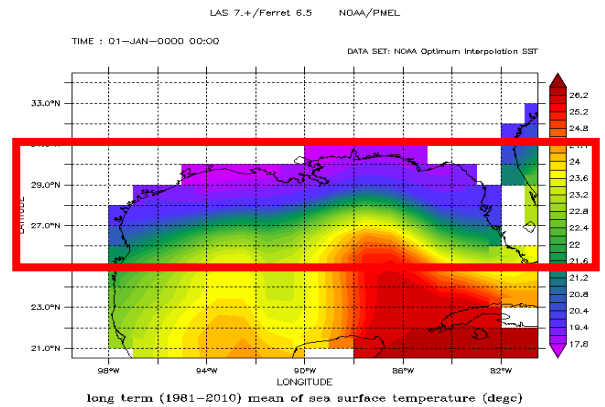
Dependency of STM-Exposure

“What does the dependency suggest and how can we overcome?”

- Why dependent?
  - *Hurricane alley*= Large STM
- Avoiding dependency
  - Clustering by K-means
  - Conditional Exposure model

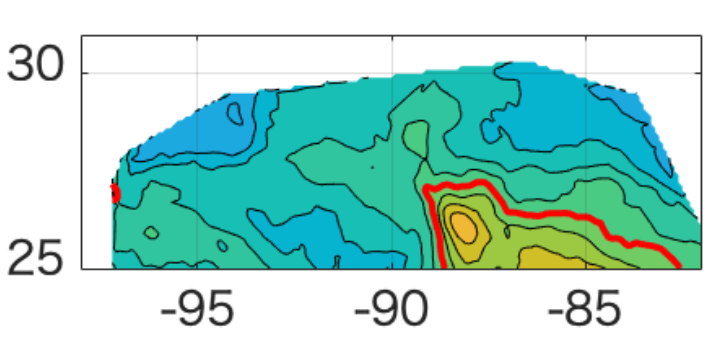


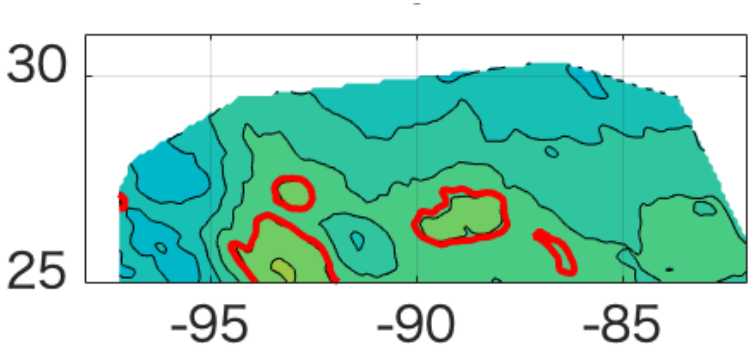
Kendall’s tau result for thresholds of 10m

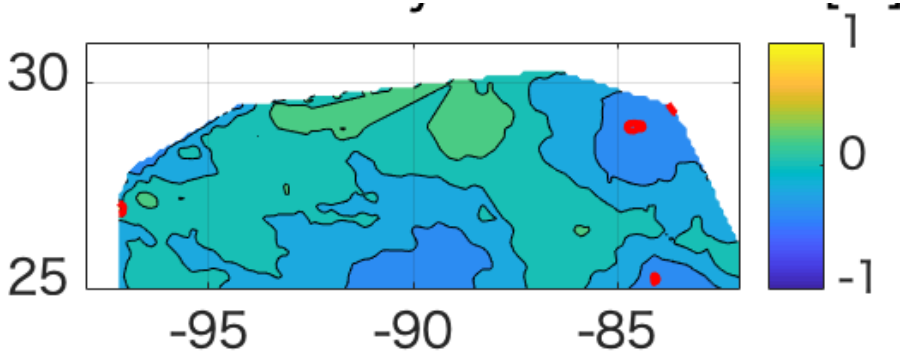


Mean SST (1981-2010) from OISST (NOAA)

Simply extracting hurricanes passing hurricane alley does not work,,,

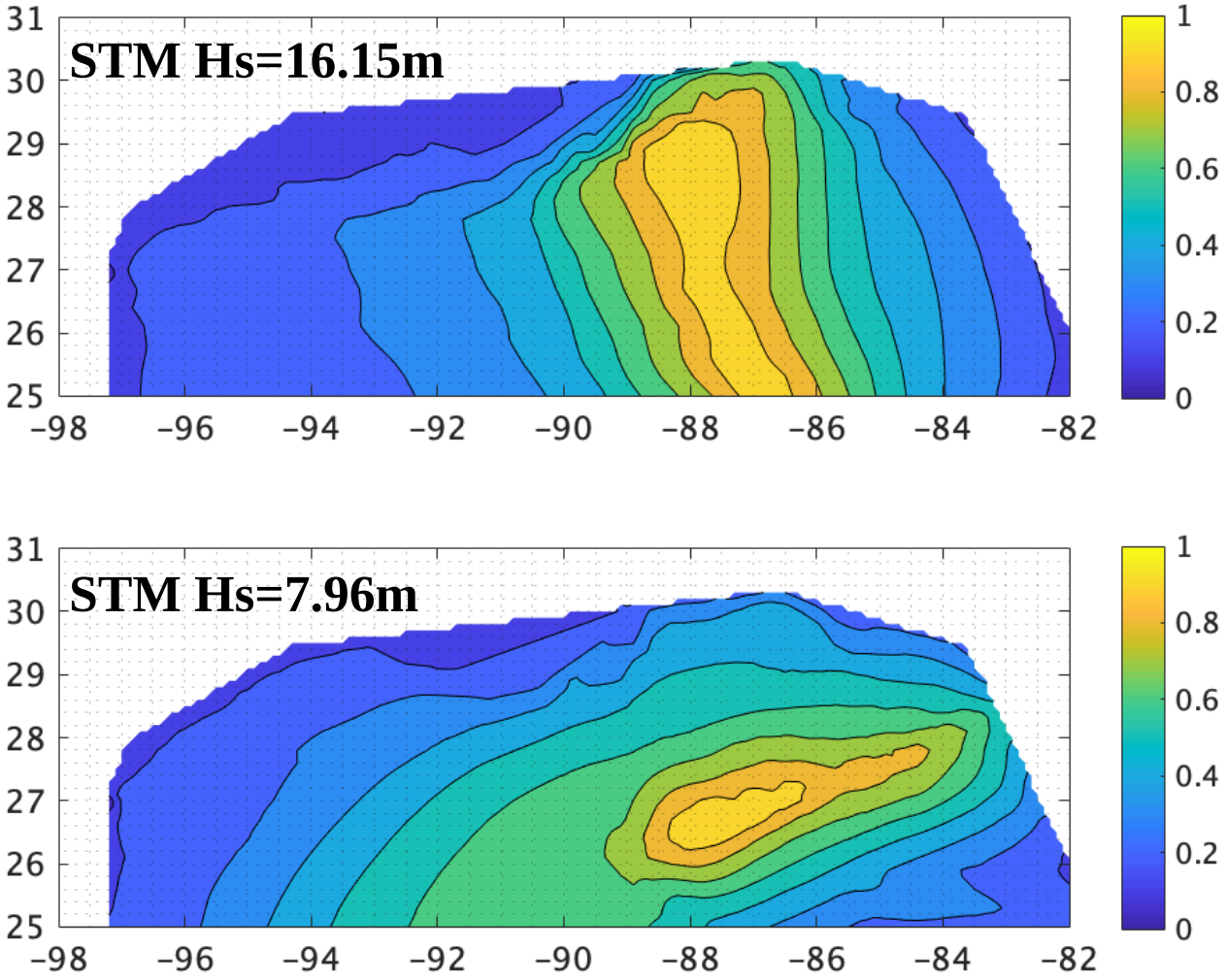




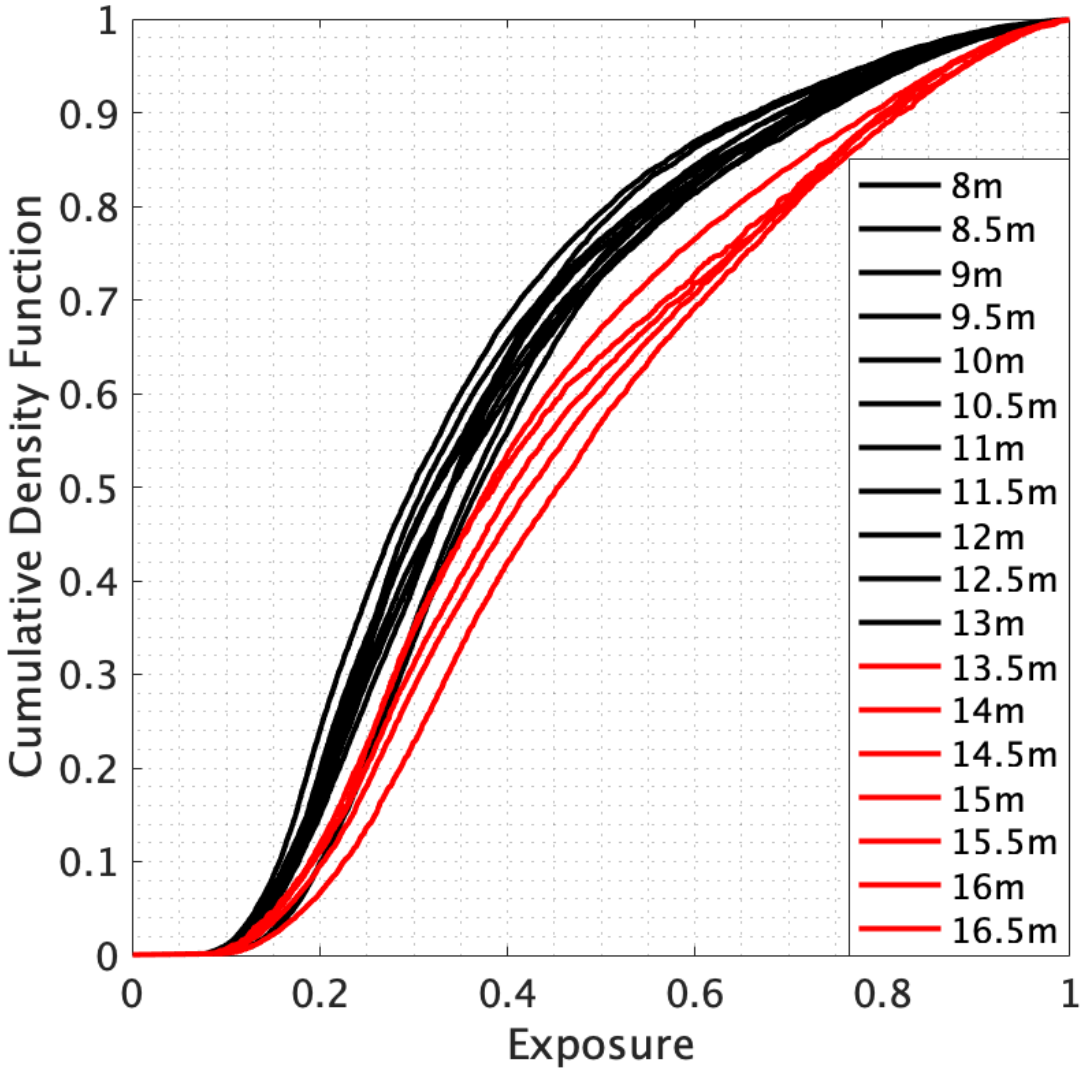


Why? What is the cause of spatial dependency?

Example of two exposure distributions

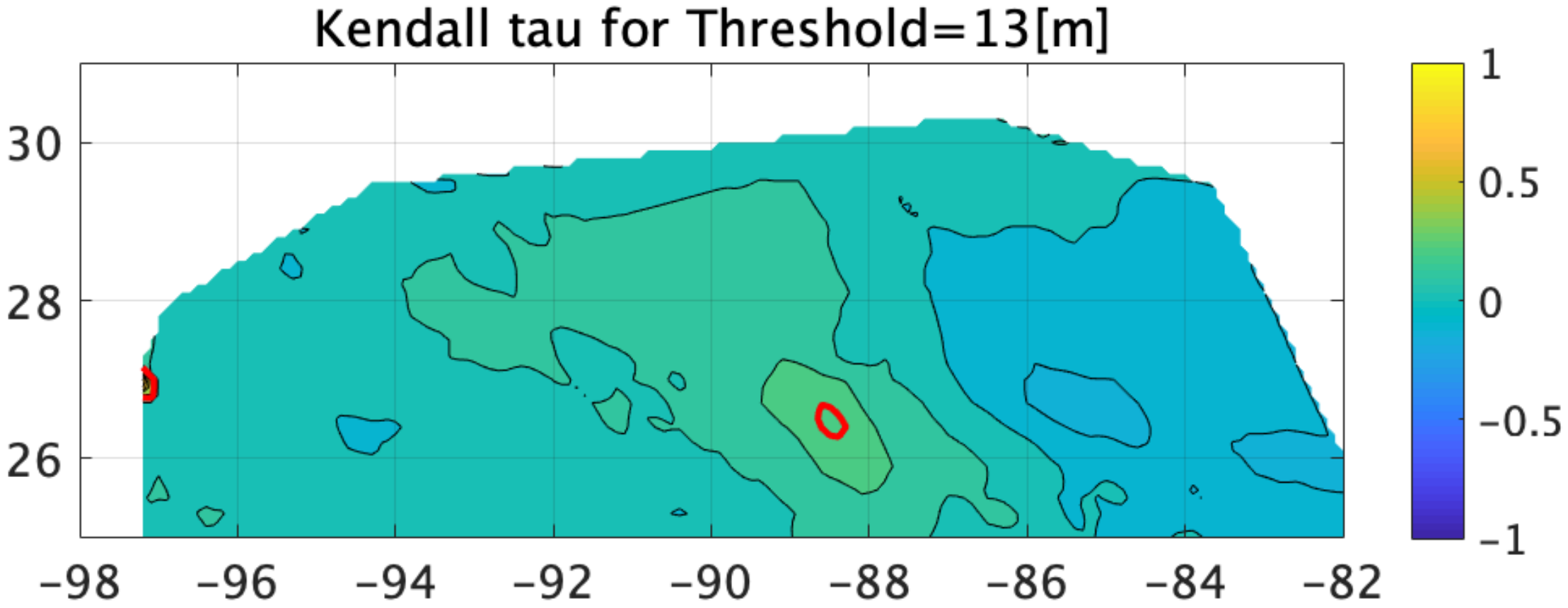


CDF of exposure with different STMs



Hurricanes with large STM (over 13m) has larger area with large exposure

Just focus on STM >13m

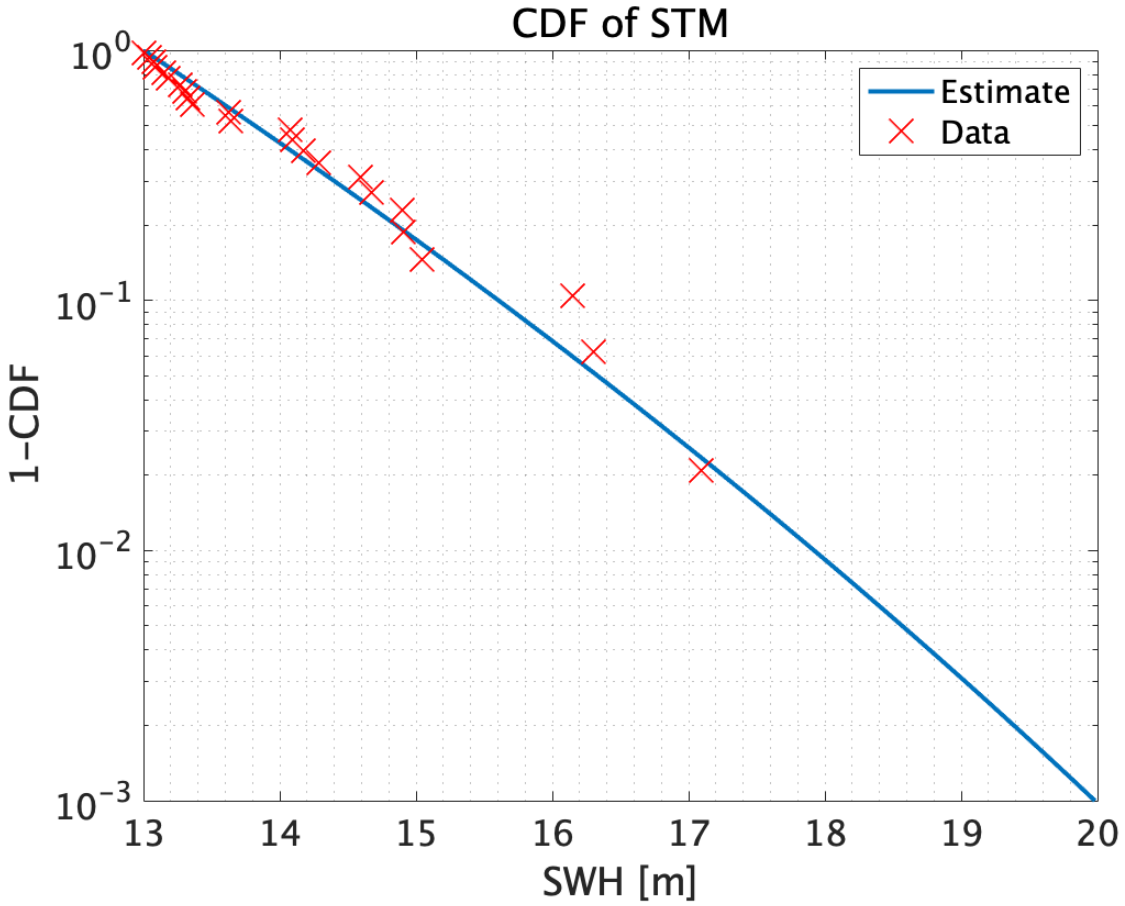


Kendall's tau result for thresholds of 13m

Considering hurricanes with large STM (Hs>13) makes STM & Exposure independent

STM for STM > 13m (N=24)

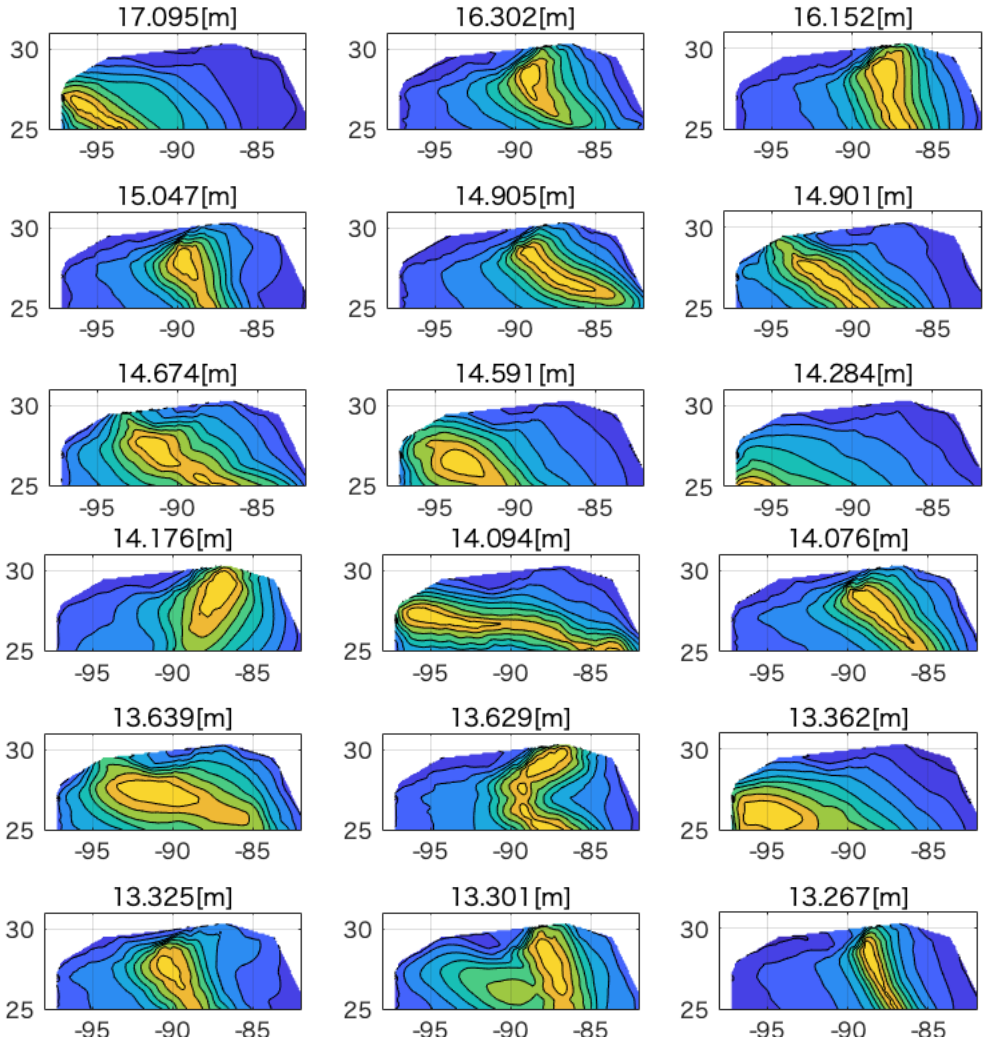
STM (Extreme value distr.) for ***Temporal extrapolation***



- STM gives the peak value for all the hurricanes in the region. Such data is not available for a per location approach. (Larger sample size)
- 100 year return period STM is around 18m. This is the expected maximum value throughout the whole region, so the extreme wave estimation for per location will be smaller.

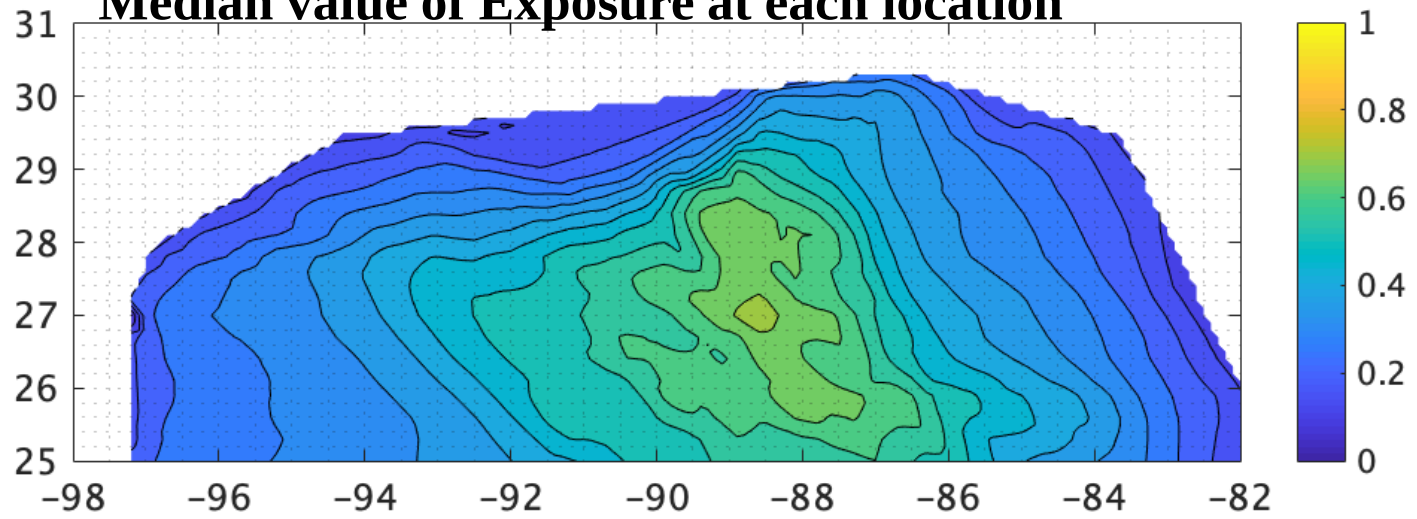
Exposure distribution for STM > 13m (N=24)

Spatial empirical interpolation

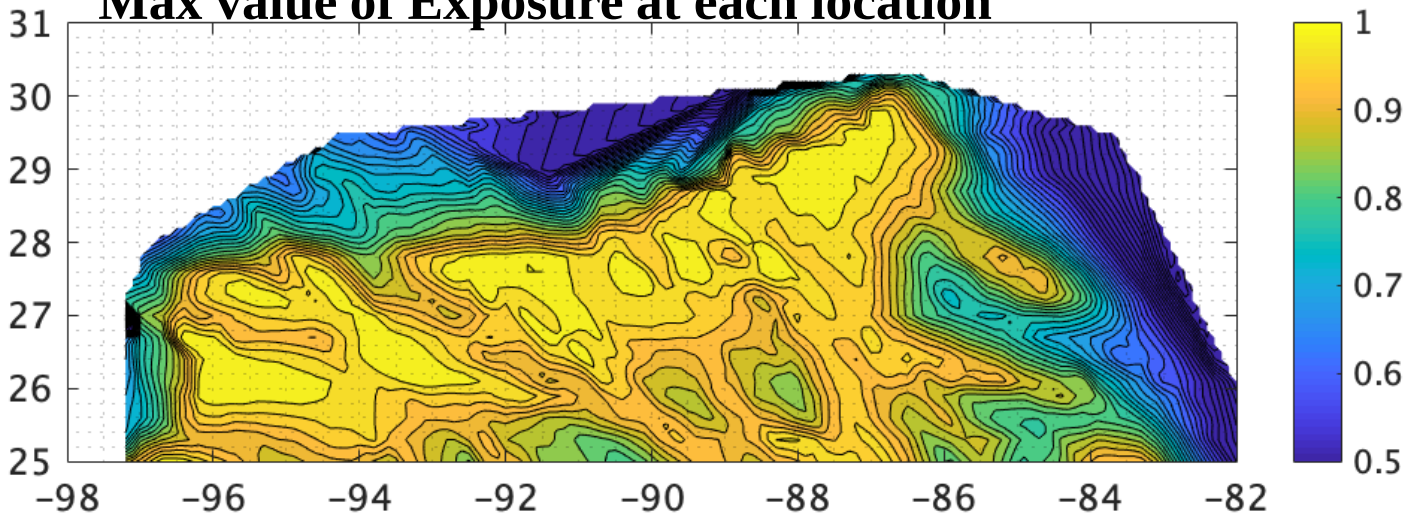


and 6 more,,,

Median value of Exposure at each location

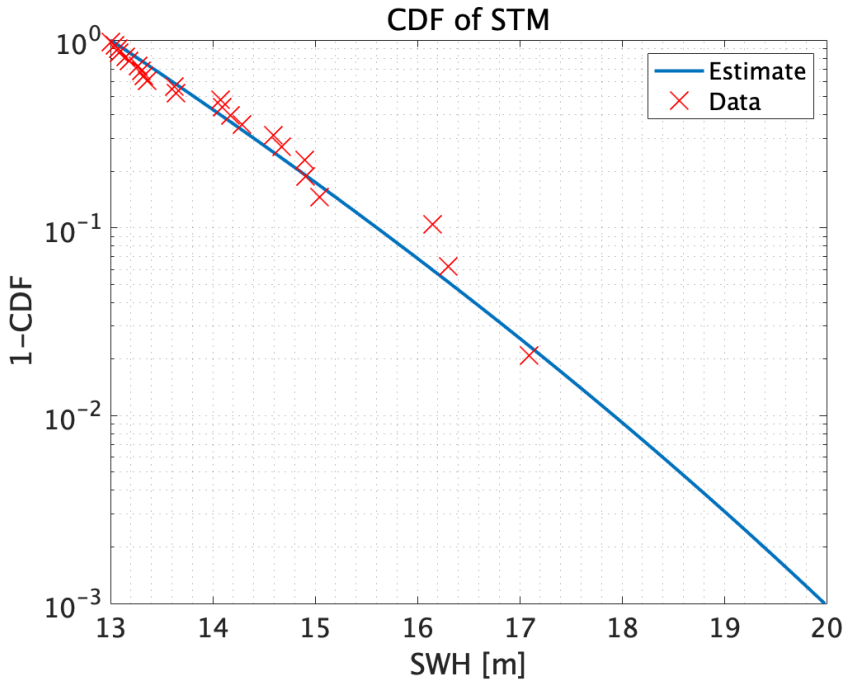


Max value of Exposure at each location

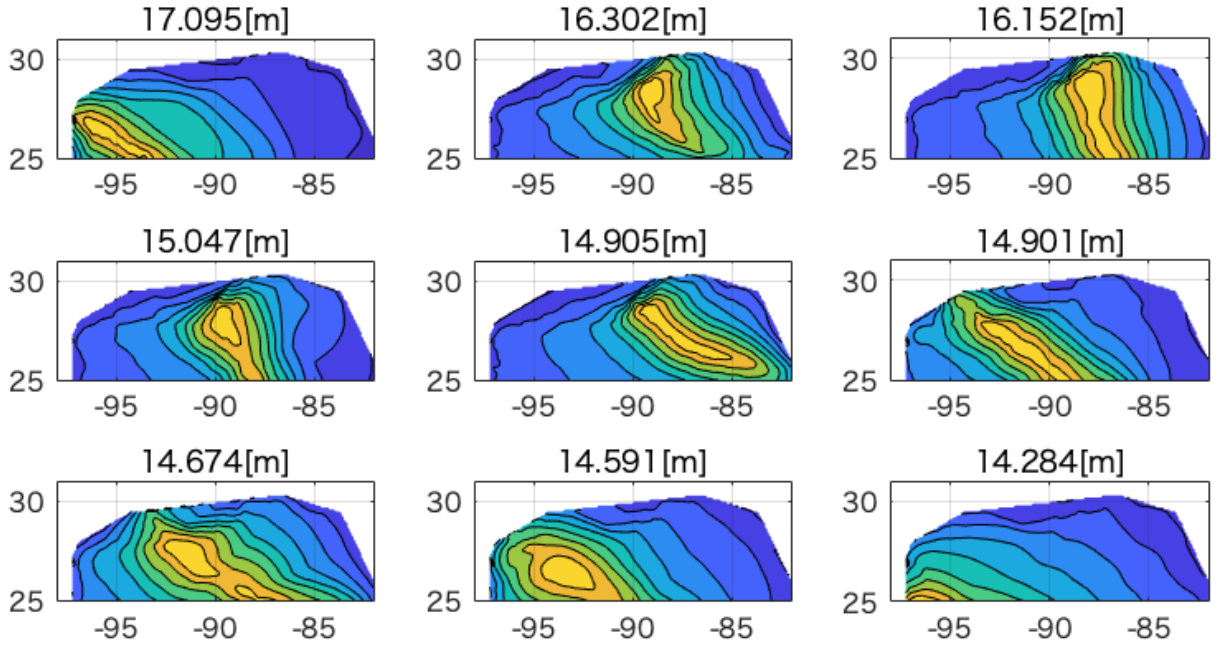


STM-E approach for STM > 13m (N=24)

STM (Extreme value distr.)



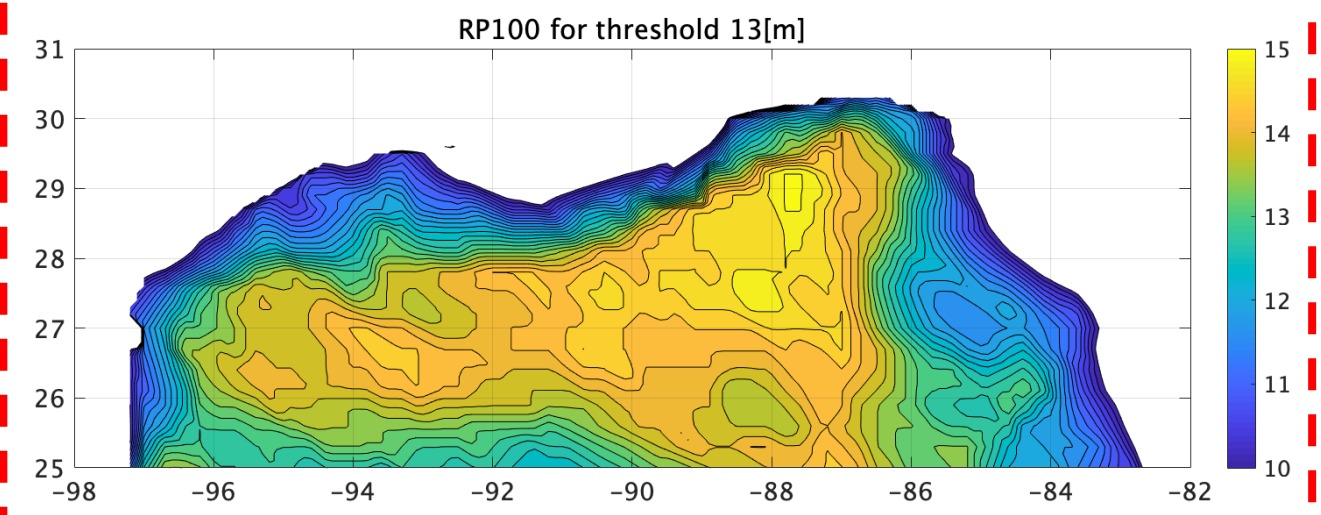
Exposure (empirical 0-1 distr.)



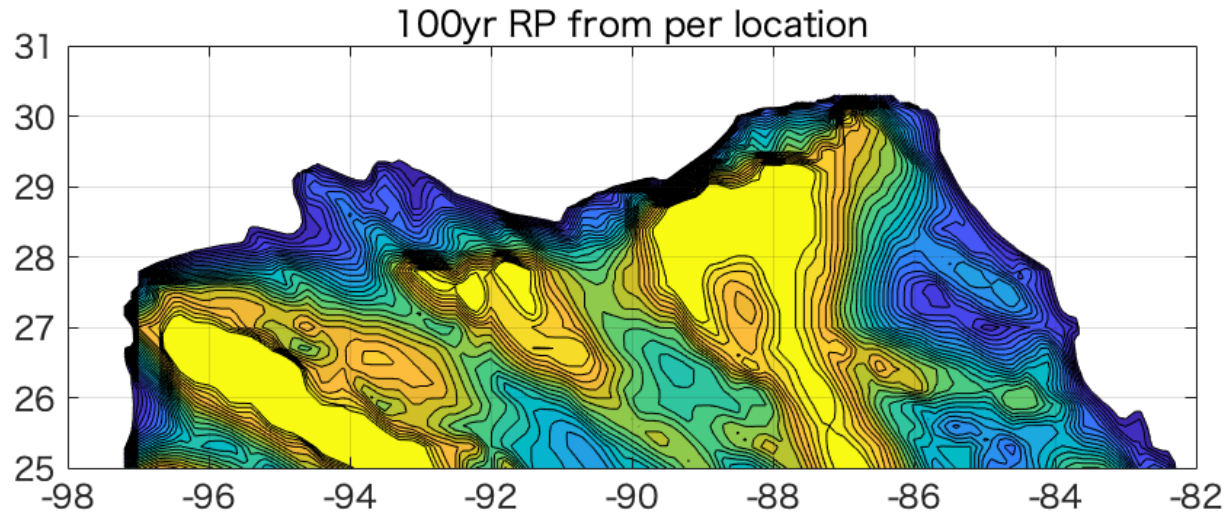
$$F_{H_j}(h) = \int_s F_{E_j}(h/s) f_S(s) ds$$

Spatial empirical interpolation **&** ***Temporal extrapolation***

Results of STM-E



100-year return values from STM-E for thresholds of 13[m]



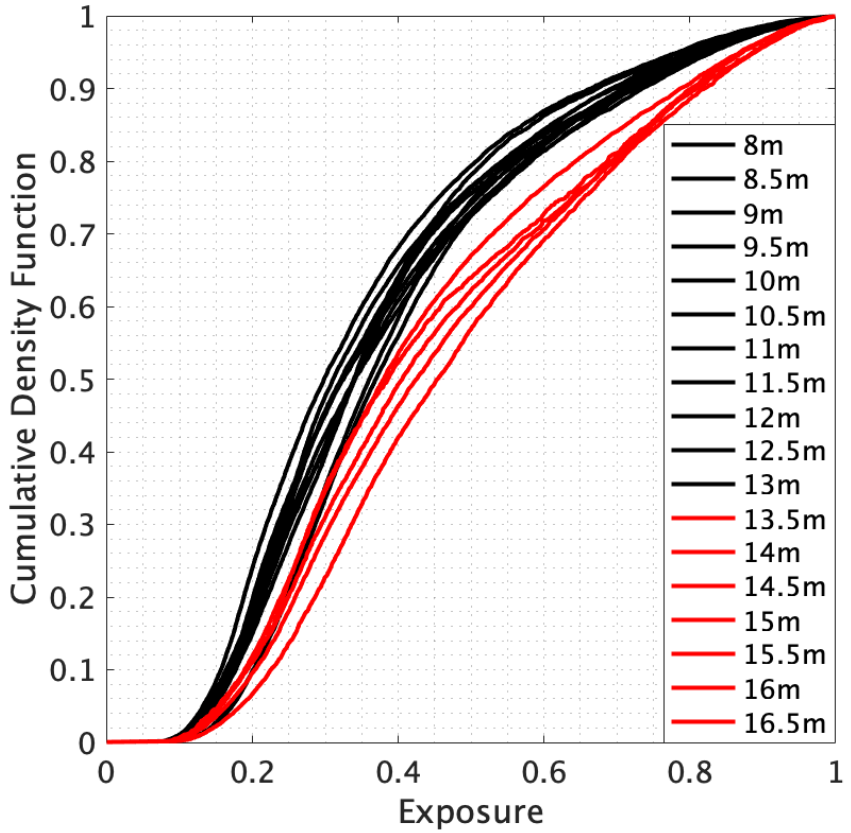
100-year return values from per location analysis

STM-E methodology applied to Gulf of Mexico provides

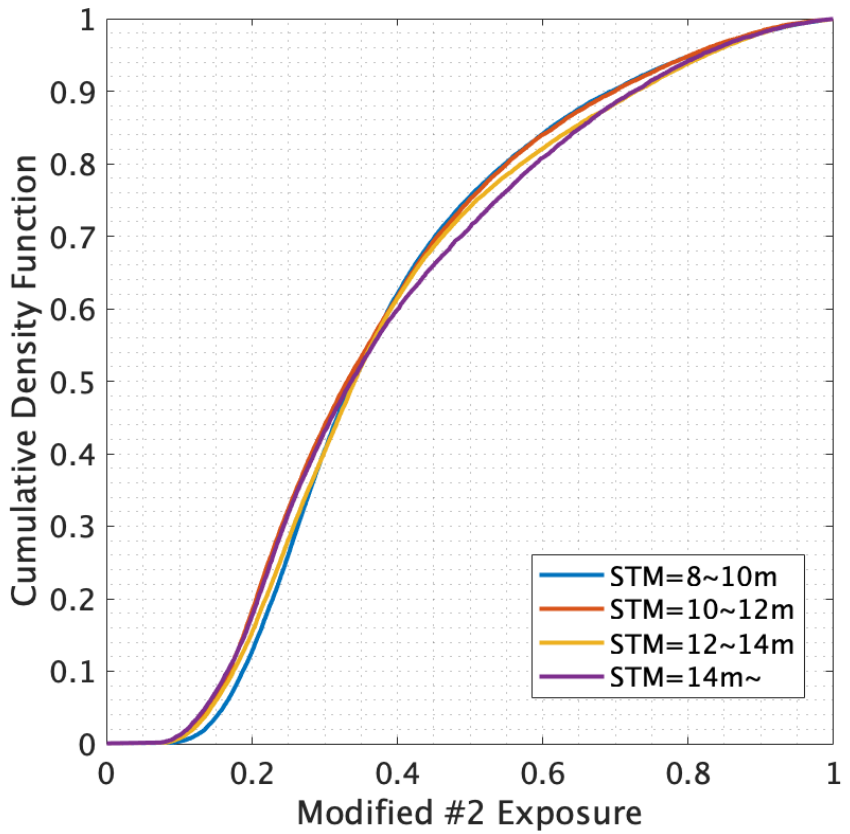
- Spatial extreme distribution based on 24 hurricanes
- Large reductions in the variability of return value estimates.
- A useful addition to the metocean engineer's toolkit.

Future work: Can we increase the number of events?

Exposure distribution differs for STM > 13m. Is it possible to use exposure distribution (track information) for STM < 13m?



Modified exposure $E' = E^{\max(STM,13)/13}$



Conclusion

“Can spatial extremes model improve extreme wave estimation in GoM?”

- STM-E provides robust and less variable extreme wave estimation in GoM.
- STM and Exposure show dependency when threshold 8, 10, 12m is considered, but is independent for STM>13m. Hurricanes with large STM has large area with large exposure.
- We are now considering whether it is possible to learn from tracks below Hs < 13m (need to consider both physics & statistics). A conditional exposure model appears to be the solution.

Acknowledgement

We thank Oceanweather for use of GOMOS data, and Dr. Andrew Cox for valuable comments on this work.

Thank you for your kind attention!

STM gives more information than per location

"STM has short tail" and "TM (=per location) has a longer tail"

STM=GP model

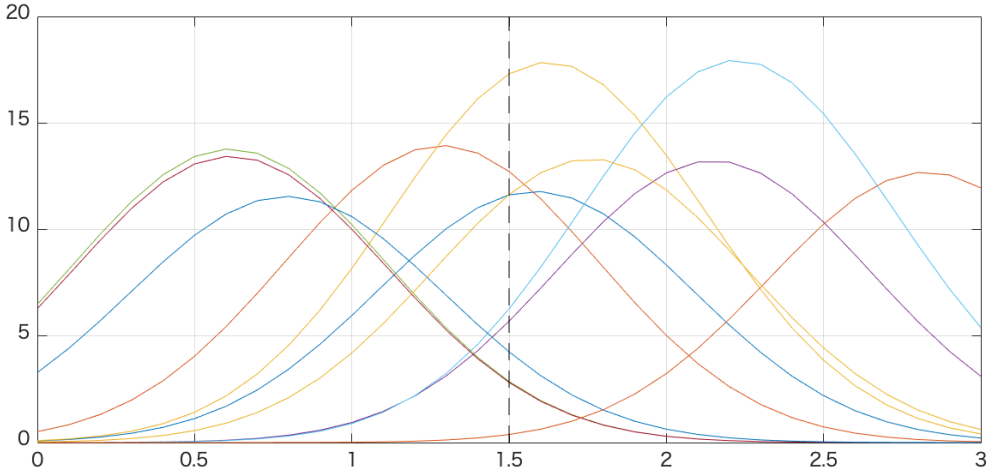
$$\xi = -0.5, \sigma = 5, \psi = 10$$

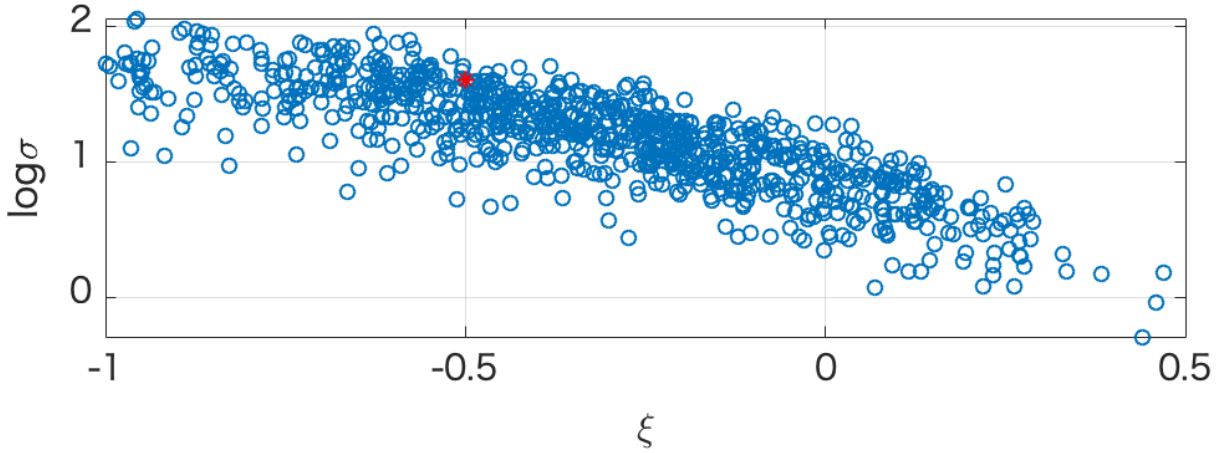
Exposure model

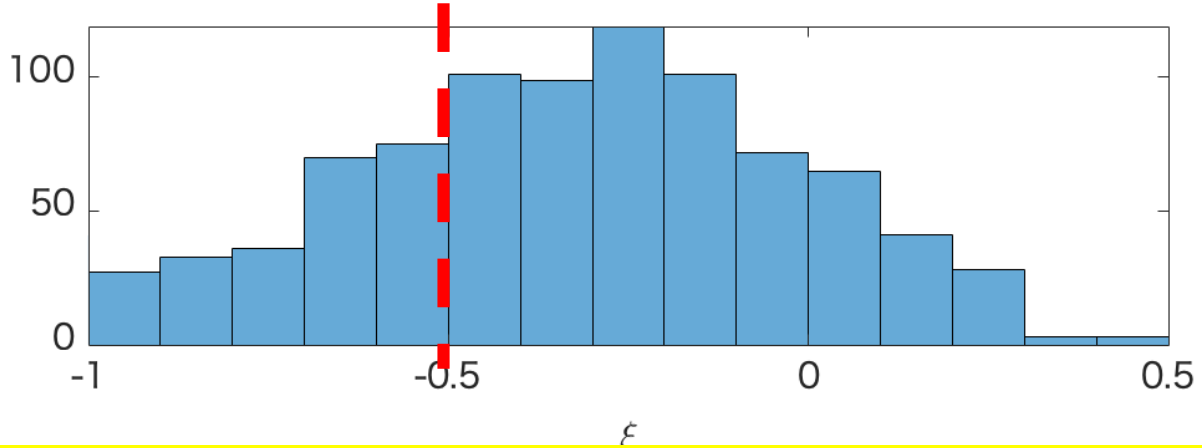
$$N(\mu, \sigma), \mu = u[0, 3], \sigma = 0.5$$

observed at x=1.5

*normalized to have max 1







"Bias can be corrected by taking a HUGE threshold"